Numerical simulation of oscillating-wing based energy harvest mechanism using the high-order spectral difference method

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A high-order spectral difference method based unsteady Navier-Stokes numerical framework is adopted to carry out simulations of oscillating-wing based energy harvest mechanism. Energy extraction with both the prescribed motion and the flow-induced motion are considered. The flow-induced motion is achieved via the fluid-structure interaction approach. Specifically, the foil dynamics is coupled into the flow solver to synchronize the flow-foil interaction. Results from both the prescribed motion approach and the flow-induced motion approach are compared. It is found that under certain conditions, the aerodynamic performances by using these two types of motions are comparable. Other aerodynamic features associated with the prescribed and the flow-induced motions are analyzed as well.

Nomenclature

\[ A \quad = \text{pitching amplitude} \]
\[ C \quad = \text{chord length} \]
\[ f \quad = \text{oscillation frequency} \]
\[ h \quad = \text{plunging amplitude} \]
\[ k \quad = \text{reduced frequency, } 2\pi f C/U_\infty \]
\[ P_{i,\theta} \quad = \text{power input to sustain the pitching motion} \]
\[ P_{o,h} \quad = \text{power output from the plunging motion} \]
\[ \bar{P} \quad = \text{time averaged net power, } 1/T \int_{t_0}^{t_0+T} (P_{o,h} - P_{i,\theta}) \, dt \]
\[ Re \quad = \text{Reynolds number based on the chord length, } \rho U_\infty C/\mu \]
\[ U_\infty \quad = \text{free stream velocity} \]
\[ Y_d \quad = \text{maximum leading/trailing edge excursion} \]
\[ \eta_{\text{power}} \quad = \text{power harvesting efficiency, } \bar{P}/(\frac{1}{2} \rho U_\infty^3 Y_d) \]
\[ \theta \quad = \text{pitching angle of the airfoil} \]
\[ \rho \quad = \text{density} \]
\[ \phi \quad = \text{phase lag between pitching and plunging motions} \]
\[ \omega \quad = \text{angular frequency of oscillation, } 2\pi f \]

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1. Introduction

In recent years, research on energy harvest from renewable sources has witnessed a surge due to the depletion of fossil fuel resources and the increasingly demanding standard for emission during the energy harvest process. Energy from sun, wind, rivers and tidal flows is arguably the cleanest and most sustainable energy on the earth. It has been recognized by some researchers [1, 2, 3, 4] that flapping-wing mechanism has the potential to extract energy from several of the aforementioned renewable sources.

About three decades ago, McKinney and DeLaurier [5] reported that the power generation efficiency from a model “wingmill” could be competitive with that from other windmill designs. The energy extraction mechanism is in analogy to flutter, which features a combined pitch and plunge motion. This indicates that the flapping wing undergoing suitable pitch and plunge motion can be used to collect energy even from uniform flow. The basic idea behind energy harvest using an oscillating foil is to align the lift and the plunge velocity in the same direction by adjusting the effective angle of attack (AOA) during most portion of an oscillation cycle. This has been clearly explained by Jones and Platzer [1]. Indeed, some practical devices using flapping-wing mechanism have been designed to harvest hydro-power, e.g. Aniprop [6], Stingray Tidal Stream Energy Device [7] and bioSTREAM\textsuperscript{TM} [8].

In order to fully understand the unsteady flow physics associated with flapping motion and to further guide the practical design, a series of numerical and experimental work have been carried out on oscillating-wing based energy extraction. These studies can be roughly classified into three categories based on the adopted kinematics, namely the prescribed motion [1, 9, 10, 11, 12], semi-prescribed motion [13] and fully flow-induced motion [14, 15]. The drawback of the prescribed motion is that it does not consider the practical design of driven mechanical devices, which can be very complex, to actuate the motion. But it is an effective tool to perform a parametric study and unveil the underlying physics of energy harvest. The fully flow-induced motion is a good model for practical use. However, there exists some uncertainty on how the motion is initially triggered. Vortex shedding can serve as one startup mechanism [16]. Some researchers [14] manually introduced a small disturbance to the system at the initial stage to expedite the flow instability. In order to alleviate the dependence on the initial condition, the semi-prescribed motion approach is adopted here to reveal the flow-foil interaction effects. In the semi-prescribed motion approach, the pitch motion is prescribed and the plunge motion is activated via fluid-structure interaction. The dynamic equation of the oscillating foil is coupled with the flow solver. As vortex dominated flow is featured in the study, a high-order spectral difference method based flow solver developed in Ref. [17] is used to carry out the numerical simulation. The high-order scheme can significantly reduce the numerical dissipation which can contaminate the small flow features.

The remainder of this paper is organized as follows. In Section II, the numerical framework, including flow-dynamics coupling procedure, is introduced. The developed flow solvers are verified in section III. Then both numerical results from the prescribed and flow-induced motions are presented and discussed in section III as well. Section IV briefly concludes the work.
II. Numerical framework

A. Governing equations

Numerical simulations are performed with an unsteady compressible Navier-Stokes solver using dynamic unstructured grid high-order spectral difference (SD) method developed in Ref. [17]. The 2D unsteady compressible Navier-Stokes equations in conservation form read,

\[ \frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0. \tag{1} \]

Herein, \( Q = (\rho, \rho u, \rho v, E)^T \) are the conservative variables, and \( F, G \) are the total fluxes including both the inviscid and viscous flux vectors, i.e., \( F = F^I - F^v \) and \( G = G^I - G^v \). The solution system is closed by assuming that the perfect gas law is obeyed.

To achieve an efficient implementation, a time-dependent coordinate transformation from the physical domain \((t, x, y)\) to the computational domain \((\tau, \xi, \eta)\) is applied to Eq. (1). And we obtain

\[ \frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial \tilde{F}}{\partial x} + \frac{\partial \tilde{G}}{\partial y} = 0, \tag{2} \]

where

\[
\begin{align*}
\tilde{Q} &= |J| Q \\
\tilde{F} &= |J| \left( Q\xi_x + F\xi_x + G\xi_x \right) \\
\tilde{G} &= |J| \left( Q\eta_x + F\eta_x + G\eta_x \right)
\end{align*}
\tag{3}
\]

Herein, \( \tau = t \) and \((\xi, \eta) \in [-1,1]^2\), are the local coordinates in the computational domain. In the transformation shown above, the Jacobian matrix \( J \) takes the following form

\[
J = \frac{\partial (x, y, t)}{\partial (\xi, \eta, \tau)} = \begin{pmatrix}
x_\xi & x_\eta & x_\tau \\
y_\xi & y_\eta & y_\tau \\
0 & 0 & 1
\end{pmatrix}.
\tag{4}
\]

Note that the grid velocity \( \vec{v}_g = (x_\tau, y_\tau) \) is related with \((\xi_\tau, \eta_\tau)\) by

\[
\begin{align*}
\xi_\tau &= -\vec{v}_g \cdot \nabla \xi \\
\eta_\tau &= -\vec{v}_g \cdot \nabla \eta
\end{align*}
\tag{5}
\]

B. Space discretization

The SD method is used for the space discretization. In the SD method, two sets of points are used, namely the solution and flux points. Conservative variables are defined at the solution points (SPs), and then interpolated to flux points to calculate local fluxes. In the present study, the solution points are chosen as the Chebyshev-Gauss quadrature points and the flux points are selected to be the Legendre-Gauss points with end points -1 and 1. Then using Lagrange
polynomials, we reconstruct all the fluxes at the flux points. Note that this reconstruction is continuous within a standard element, but discontinuous on the cell interfaces. Therefore, for the inviscid flux, a Riemann solver is necessary to reconstruct a common flux on the interface. The reconstruction of the viscous flux is based on the average of the ‘left’ and ‘right’ fluxes. The detailed reconstruction procedures can be found in Ref. [17].

C. Kinematics and simulation parameters

Two types of kinematics, namely the prescribed motion and the flow-induced motion will be studied here. As indicated in Refs. [4], in the flow-induced motion approach, power is mainly extracted from the plunging motion. The pitching motion can be actively controlled or passively excited by flow-induced instability. In order to alleviate the uncertainty from flow-induced instability in the passively pitching approach, only actively controlled pitching motion is studied here.

C.1 Prescribed motion approach

The kinematics of the foil is specified as follows.

\[ \text{Plunge motion: } y = h \sin(\omega t), \]
\[ \text{Pitch motion: } \theta = A \sin(\omega t + \phi), \]  

where \(y\) and \(\theta\) are the plunge displacement and pitch angle of the airfoil respectively, \(h\) and \(A\) are the plunge and pitch amplitude respectively, \(\omega\) is the angular frequency of oscillation and \(\phi\) is the phase lag between the plunge and pitch motions.

As illustrated in Ref. [1], when the geometric AOA exceeds the plunge-induced AOA, the lift can do work on the oscillating foil. This work can be collected via certain mechanical system, like a damper [13, 14]. The generated power is then expressed as \(P_{h} = \text{Lift} \times \dot{y}\). The external torque used to actuate the pitch motion is \(M_{a} = -M_{\text{pit}}\). Note that it is possible that the aero-/hydro-dynamic moment \((M_{\text{pit}})\) can also extract energy from the pitching motion. In this case, the pitch motion is self-sustained, and in order to maintain the prescribed sinusoidal motion, complicated control and activation system might be necessary. Then the power input to sustain the pitch motion can be expressed as \(P_{i, \theta} = -M_{\text{pit}} \times \dot{\theta}\) (Note that if \(P_{i, \theta}\) is negative, then power is in fact harvested from the flow via the pitch motion). The mean net power output is then written as \(P = 1/T \int_{t_{0}}^{t_{0}+T} (P_{h} - P_{i, \theta}) dt\). The power harvest efficiency is defined by the ratio between net power output and the total kinematic energy in the region swept by the foil, i.e. \(\eta_{\text{power}} = P / (1/2 \rho U_{\infty}^{2} Y_{d})\), where \(Y_{d}\) is the maximum leading/trailing edge excursion.

C.2 Fluid-structure interaction approach

In the fluid-structure interaction approach, the pitch motion is prescribed as in Eq. (6). Then the plunge motion is triggered by the unsteady force exerted on the foil, and the energy is
harvested via a damper with damping coefficient $c$. The dynamic equation for the plunge motion without stiffness reads

$$m_s \ddot{y} + c \dot{y} = F_y.$$  \hspace{1cm} (7)

Herein, $m_s$ is the mass of the solid and $F_y$ is the aerodynamic force in the $y$ direction. In the present setup, $F_y$ is actually equivalent to the lift.

The coupling of the foil dynamics and fluid equations is enforced at each time step. In order to save computational cost, when a Runge-Kutta time integration is used, the aerodynamic force and moment are only calculated at the last stage. Then the lift force is passed to the foil and the plunging velocity of the foil is calculated from Eq. (7). This velocity combined with that from the prescribed pitching motion is passed back to the flow solver for grid deformation. The whole process is then repeated for the next time step.

One major challenge from the partitioned fluid-structure interaction algorithm using conforming meshes is the proper treatment of the fluid-solid interface. When the density ratio $\rho_s/\rho_f$ between the solid and fluid is small, the added-mass instability will ruin the numerical simulation very quickly if the interface data transfer is not carefully handled. Many methods to suppress this instability have been developed for the incompressible flow [18]. In this study, as a low Mach number preconditioned compressible solver [17] is used, a linearized fluid-solid Riemann problem [19] is solved to weakly enforce the interface conditions,

$$\begin{align*}
\mathbf{n} \cdot \mathbf{v}^s &= \mathbf{n} \cdot \mathbf{v}^f, \\
\mathbf{n} \cdot \mathbf{\sigma}^s &= \mathbf{n} \cdot \mathbf{\sigma}^f.
\end{align*}$$

Herein, the superscript ‘$s$’ indicates solid and ‘$f$’ indicates fluid. $\mathbf{v}$ and $\mathbf{\sigma}$ stand for the velocity vector and stress tensor on the interface, respectively. $\mathbf{n}$ stands for the normal direction of the interface. It is found that this approach can mitigate the added-mass instability effectively. A strong coupling algorithm using the predictor-corrector approach [20] is used to further enhance the stability of the simulation. As explicit time integration is adopted in the present study, only one correction step is carried out during the simulation. It turns out that this iteration strategy works well for all studied cases involving fluid-structure interaction.

III. Results and discussions

A. Flow solver verification

The flow-induced vibration of a zero-mass cylinder is simulated to test the capability of the present solver. The cylinder diameter based Reynolds number $Re_D$ for this case is set as 100 and the far field boundary is located 50 diameters away from the cylinder. The results from $h$-refinement (grid refinement) and $p$-refinement (polynomial degree) studies are presented in Table 1. The computed parameters include the normalized vortex-induced plunge amplitude, frequency and the time averaged thrust coefficients. Comparison of these parameters with those from previous studies [21, 22, 13, 18] is summarized in the same table as well. It is concluded that the present results agree well with the results from literatures.
In order to determine the suitable grid and scheme accuracy for the simulations of the oscillating airfoils, \( hp \)-refinement studies are carried out using a fluid-solid interaction case at \( Re = 1,000, k = 1.0 \) and \( A = 70^\circ \). The density ratio \( \rho_s/\rho_f \) is set as 4.4 and the normalized damping coefficient \( c' \) is 3.6. From the results as shown in Fig. 1, it is clear that the variations of lift coefficients within one oscillation cycle for the 3rd order scheme with both coarse and fine grids and the 4th order scheme with the coarse grid agree well with each other, while that for the 2nd order scheme with the coarse mesh shows marked deviation. Based on these results, the 3rd order accurate SD scheme with a coarse mesh is chosen to carry out the simulations. Moreover, the non-dimensional time step \( (\Delta t = \Delta t/U_\infty/C) \) used in the simulations is set as \( 1.0 \times 10^{-4} \), which is determined from a time refinement study on the case with the chosen accuracy and grid configuration.

B. Prescribed motion

The results with prescribed pitch and plunge motion are discussed in this section. As previously mentioned, this serves as a parametric study for searching optimal flapping kinematics. The chord length based Reynolds number is fixed at 1,000. Based on the studies in Ref. [9, 4], three relatively small reduced frequencies, namely 0.8, 1.0 and 1.2, are chosen. Then different combinations of the normalized plunge amplitude \( h/C \) and the pitch amplitude \( A \) are tested and results are summarized in Fig. 2. Several trends are concluded as follows:

- At the same reduced frequency, the moderate pitch amplitude (e.g. \( 70^\circ \)) with relatively large normalized plunge amplitudes (e.g. 0.5 and 1.0) is conducive for maintaining high energy extraction efficiency. This observation agrees with previous results [9, 4].
- With a relatively small pitch amplitude (e.g. \( 50^\circ \)), the large plunge amplitude (e.g. 1.0) is not good for energy extraction. This is due to the fact that this parameter combination approaches the feathering limit [9] of energy extraction as the reduced frequency increases.

Two cases with different energy harvest performance are then compared in details regarding the energy extraction histories and flow fields. Parameters for the two cases are \( k = 1.0, h/C = 1.0, A = 50^\circ \) and \( k = 1.0, h/C = 0.5, A = 70^\circ \). The first case features a relatively large plunge amplitude and small pitch amplitude. The energy harvest efficiency is only about 1.8%. The second case features a moderate plunge amplitude and moderate pitch amplitude with the energy harvest efficiency of 33.2%. The histories of the overall energy harvest efficiency and contributions from plunge and pitch motions are displayed in Fig. 3. From Fig. 3(a), it is observed that not much net energy is extracted by the plunge motion and the pitch motion is consuming energy during the entire process. Therefore, the overall energy extraction performance is not satisfactory. In contrast, it is found from Fig. 3(b) that with suitable kinematic parameters, large amount of net energy is collected by the plunge motion and the energy consumption by the pitch motion is very small. All these aerodynamic performances are closely related to the flow fields.
The vorticity fields at four different phases, namely 0, \( \pi/2 \), \( \pi \) and \( 3\pi/2 \) in an oscillating cycle are shown in Fig. 4 and Fig. 5 for both cases. It is observed that for the case with inferior energy harvest performance, there does not exist leading edge separation during the entire oscillation cycle. But for the case with promising energy harvest performance, large leading edge vortices are formed during the oscillation. This can induce a low pressure region on the top (bottom) surface of the airfoil in the upstroke (downstroke), favoring the upward (downward) lift production. As the direction of the lift force coincides with that of the plunge velocity during most portion of the oscillation, large lift can enhance the energy harvest performance.

C. Fluid-structure interaction

In this section, the energy extraction performance via the fluid-structure interaction approach is discussed. The chord length based Reynolds number is fixed at 1,000. According to Ref. [13], the normalized damping coefficient \( c' \) is chosen as \( \pi \), at which high performance in terms of energy generation has been reported. The density ratio \( \rho_s/\rho_f \) is set as 4.0 for all simulations. Similar to the prescribed motion, three relatively small reduced frequencies, namely 0.8, 1.0 and 1.2, are chosen. Then different pitch amplitudes are tested. The flow-induced plunge amplitudes and energy harvest efficiencies are summarized in Table 2. Several observations are summarized as follows.

- All the cases show satisfactory energy harvest performance except that with \( k = 1.2 \) and \( A = 90^\circ \).
- The moderate reduced frequency (e.g. 1.0) is more beneficial for maintaining high energy harvest efficiency compared with the lower and higher reduced frequencies. The largest energy harvest efficiencies at all three reduced frequencies appear at the moderate pitch amplitude (e.g. \( 70^\circ \)).
- All normalized induced plunge amplitudes do not exceed 0.5 with \( c' = \pi \). By revisiting the feather limit of energy harvest in Ref. [9], it is found that this phenomenon actually ensures that the kinematic parameters are well away from the feather limit to guarantee effective energy extraction.

The histories of the plunge traces with different pitch amplitudes at the three reduced frequencies are displayed in Fig. 6. Two cases with different energy harvest performance are then compared in details regarding the energy extraction histories and flow fields. Parameters for the two cases are \( k = 1.0, A = 70^\circ \) and \( k = 1.2, A = 90^\circ \). Large energy harvest efficiency is obtained in the first case, while relatively small one in the second case. The histories of the overall energy harvest efficiency and contributions from plunge and pitch motions are displayed in Fig. 7. Compared with the results from the prescribed motion as shown in Fig. 3, several observations are concluded as follows.

- No matter the overall energy harvest efficiency is large or not, the induced plunge motion always extracts energy from the flow. This is the intrinsic feature behind the specified energy harvest mechanism.
The energy harvest history of the case with $k = 1.0$, $A = 70^\circ$ via the fluid-structure interaction approach is very similar to that of the case with $k = 1.0$, $h/C = 0.5$, $A = 70^\circ$ via the prescribed motion approach. As a matter of fact, the normalized plunge amplitude of the flow-induced case is 0.4, which is close to that in the prescribed motion case.

By comparison between these two cases in Fig. 7, it is found that the energy consumption properties of the pitch motion are very different. This results in significantly different energy harvest performances of the two cases. The vorticity fields at four different phases, namely $0$, $\pi/2$, $\pi$ and $3\pi/2$ in an oscillating cycle are shown in Fig. 8 and Fig. 9 for both cases. The vortex structures are similar for these two cases, although they yield different energy harvest efficiencies.

IV. Conclusions

A high-order accurate SD scheme based unsteady Navier-Stokes flow solver is used to carry out simulations of the oscillating-wing based energy harvest mechanism. Two types of kinematics, namely the prescribed motion and the flow-induced motion, are adopted in the simulations. In the fluid-structure interaction approach, the foil dynamics is coupled into the flow solver to synchronize the flow-foil interaction. The fluid-solid coupling algorithm is verified using the flow-induced vibration of a zero-mass cylinder. A parametric study is carried out by adopting the prescribed motion. It is found that the differences of aerodynamic performances exhibited by various cases studied are closely related to the formation of leading edge vortices. Then results from the flow-induced motion approach are compared with those from the prescribed motion approach. It is found that under certain conditions, the aerodynamic performance of both types of motions is comparable. Specifically, the energy harvest efficiencies of both types of motions can achieve more than 30%.

References


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<th>$fC/U_\infty$</th>
<th>$\bar{C}_d$</th>
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<td>0.156</td>
<td>1.73</td>
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<tr>
<td>Eldredge [22]</td>
<td>0.45</td>
<td>0.148</td>
<td>1.66</td>
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<td>Zhu and Peng [13]</td>
<td>0.47</td>
<td>0.175</td>
<td>1.75</td>
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<tr>
<td>Yu, Baek &amp; Karniadakis [18]</td>
<td>0.42</td>
<td>-0.155</td>
<td>-</td>
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Table 1. Comparison of the normalized plunge amplitude, frequency and time-averaged drag coefficients of a zero-mass oscillating cylinder driven by vortex shedding at $Re_D = 100$.

Fig. 1. Lift coefficients from $hp$-refinement studies for the fluid-solid interaction case.
Fig. 2. The energy harvesting efficiency as a function of the prescribed plunge amplitude with different prescribed pitch amplitudes at (a) \( k = 0.8 \), (b) \( k = 1 \) and (c) \( k = 1.2 \).
Fig. 3. Histories of the overall energy harvesting efficiency and contributions from plunge and pitch with the prescribed motion at (a) $k = 1, h/c = 1, A = 50^\circ$ and (b) $k = 1, h/c = 0.5, A = 70^\circ$.

Fig. 4. Spanwise vorticity fields with $k = 1, h/c = 1, A = 50^\circ$ at different phases with the prescribed motion.
Fig. 5. Spanwise vorticity fields with $k = 1$, $h/c = 0.5$, $A = 70^\circ$ at different phases with the prescribed motion.

<table>
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<th>Pitch amplitude ($A$)</th>
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<td>$A = 50^\circ$</td>
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<td>$0.25$</td>
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<td></td>
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Table 2. Plunge amplitudes and energy harvesting efficiencies of flow-driven mechanism at different reduced frequencies and pitch amplitudes.
Fig. 6. Histories of the plunge traces with different pitch amplitudes for the flow-driven mechanism at (a) \( k = 0.8 \), (b) \( k = 1 \) and (c) \( k = 1.2 \).
Fig. 7. Histories of the overall energy harvesting efficiency and contributions from plunge and pitch for the flow-driven mechanism at (a) \( k = 1, A = 70^\circ \) and (b) \( k = 1.2, A = 90^\circ \).

Fig. 8. Spanwise vorticity fields with \( k = 1, A = 70^\circ \) at different phases for the flow-driven mechanism.
Fig. 9. Spanwise vorticity fields with $k = 1.2$, $A = 90^\circ$ at different phases for the flow-driven mechanism.